Physics I

Lecture 10

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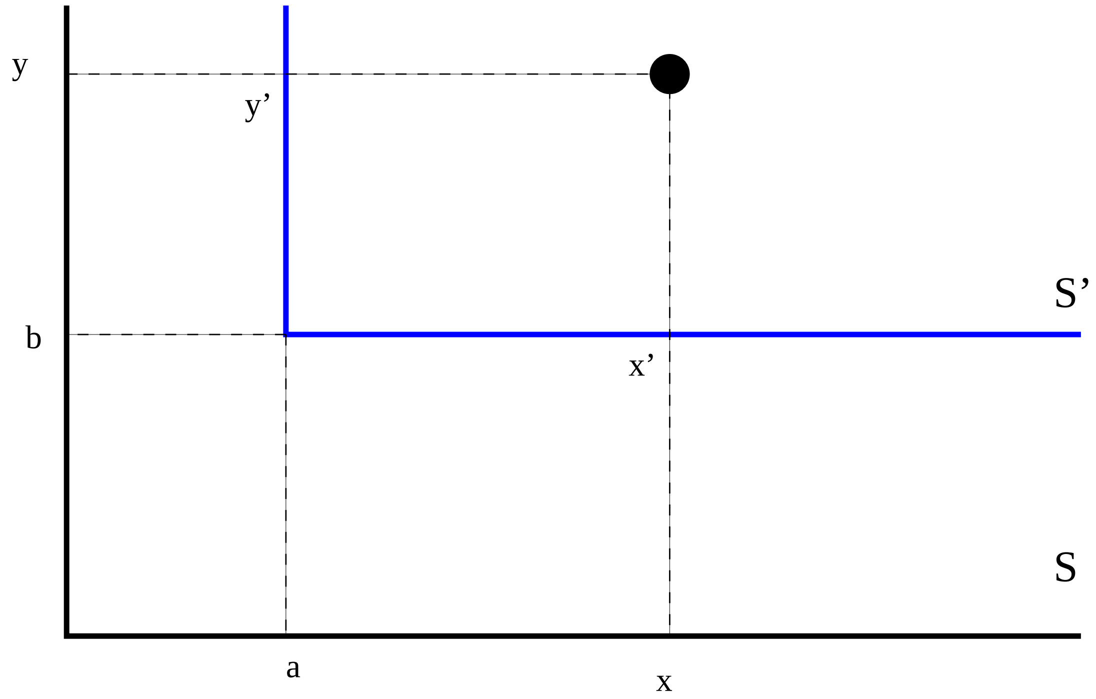
Coordinate Systems

- In 2D: Each point of the space is represented by a unique pair of numbers.
- There are infinite ways in which a coordinate system could be setup.
- The relationship between two sets is called as coordinate transformation.

Example



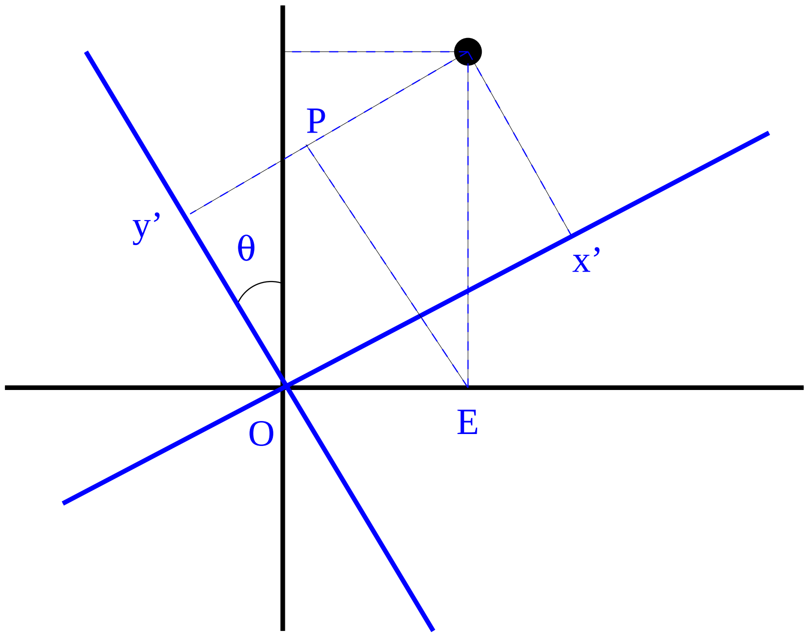
Coordinate transformations

$$
\begin{aligned}
x &= a + x' \\
y &= b + y'
\end{aligned}
$$

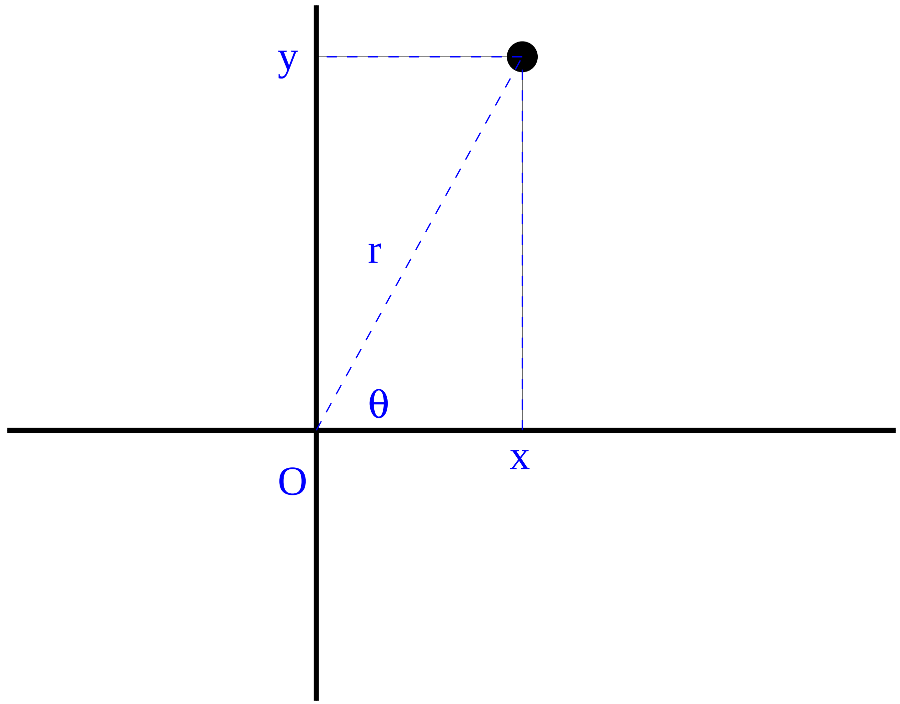
Example

Coordinate transformations

$$x' = \mathrm{OE}\cos\theta + \mathrm{EP}\sin\theta$$
$$= x\cos\theta + y\sin\theta$$
$$y' = -\mathrm{OE}\sin\theta + \mathrm{EP}\cos\theta$$
$$= -x\sin\theta + y\cos\theta$$



Example



Coordinate transformations

$$
\begin{aligned}
r &= \sqrt{x^2 + y^2} \\
\theta &= \tan^{-1}\left(\frac{y}{x}\right)
\end{aligned}
$$

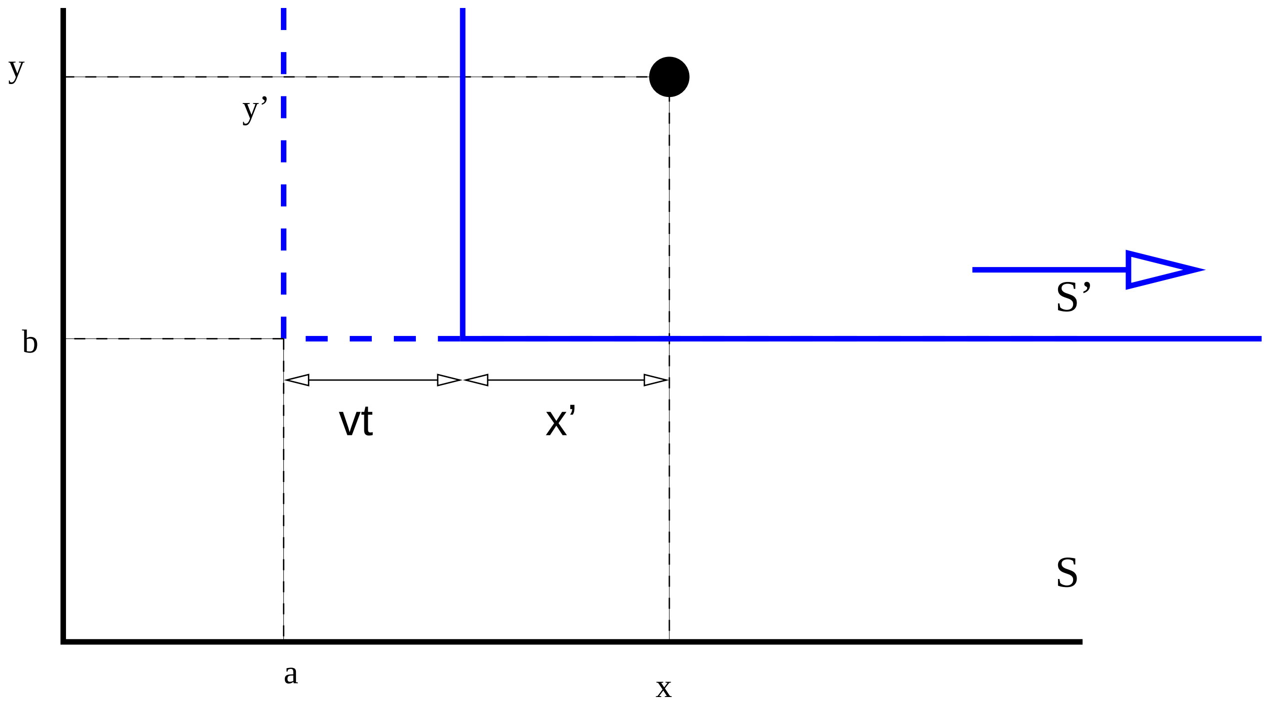
Frame of Reference

- An observer with a coordinate system and a clock etc.
- Could be moving!
- Coordinate transformations may be time dependent

Example

Coordinate Transformations

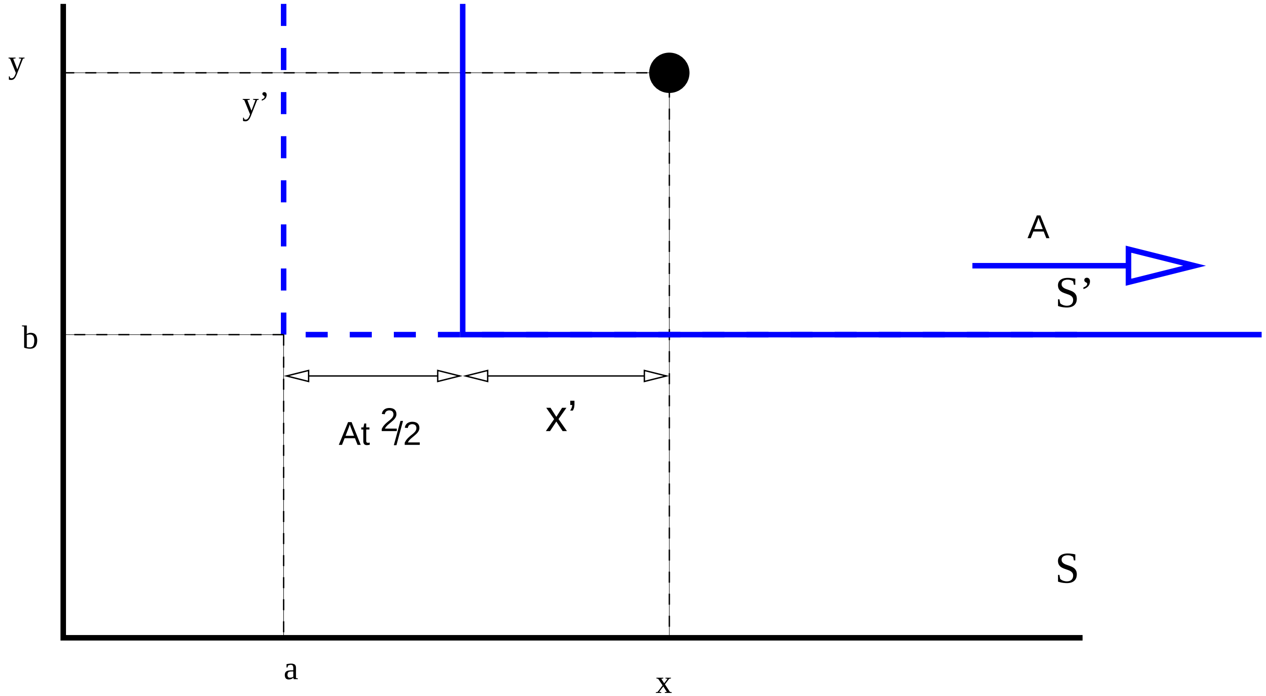
$$
\begin{aligned}
x' &= x - a - vt \\
y' &= y - b
\end{aligned}
$$



Example

Coordinate Transformations

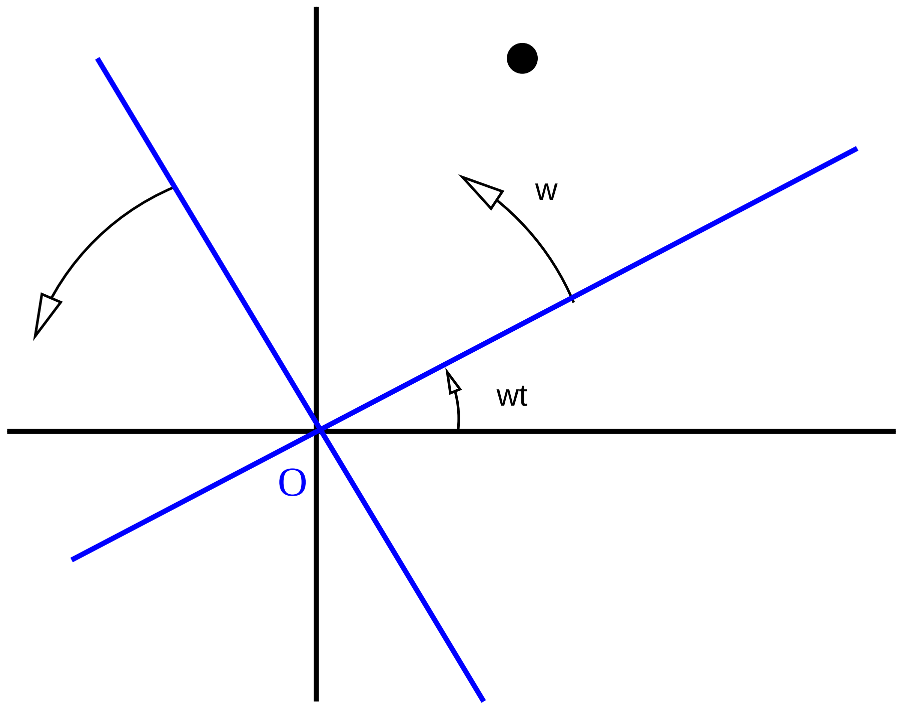
$$x' = x - a - \frac{1}{2}At^2$$
$$y' = y - b$$



Example

Coordinate Transformations

$$
\begin{aligned}
x' &= x\cos(\omega t) + y\sin(\omega t) \\
y' &= -x\sin(\omega t) + y\cos(\omega t)
\end{aligned}
$$



Inertial Frames

If S' is moving wrt S with velocity V. In S frame a particle is moving under influence of a force F. The coordinate transformations are

$$\vec{r}' = \vec{r} - \vec{V}t$$

The velocity and acceleration of the particle

$$\begin{aligned} \vec{u}' &= \vec{u} - \vec{V} \\ \vec{a}' &= \vec{a} \end{aligned}$$

Accelerations measured in two frames are same!

Inertial Frames

The force measured in S' be F'. Is F' = F?

Consider electrostatic force between two particles.

$$\vec{F} = k\frac{q^2}{|\vec{r}_1 - \vec{r}_2|^3}(\vec{r}_1 - \vec{r}_2)$$

But since $(\vec{r}_1' - \vec{r}_2') = (\vec{r}_1 - \vec{r}_2)$, $F' = F$.

Inertial Frames

If S is inertial, then

$$\begin{aligned} m\vec{a} &= \vec{F} \\ \Rightarrow m\vec{a}' &= \vec{F}' \end{aligned}$$

S’ must be inertial! (Galilean Relativity)

Uniformly Accelerating Frames

If S' is moving wrt S with acceleration A. In S frame a particle is moving under influence of a force F. The coordinate transformations are

$$\vec{r}' = \vec{r} - \frac{1}{2}\vec{A}t^2$$

The velocity and acceleration of the particle

$$\begin{aligned} \vec{u}' &= \vec{u} - \vec{A}t \\ \vec{a}' &= \vec{a} - \vec{A} \end{aligned}$$

Accelerations measured in two frames are not same!

Uniformly Accelerating Frames

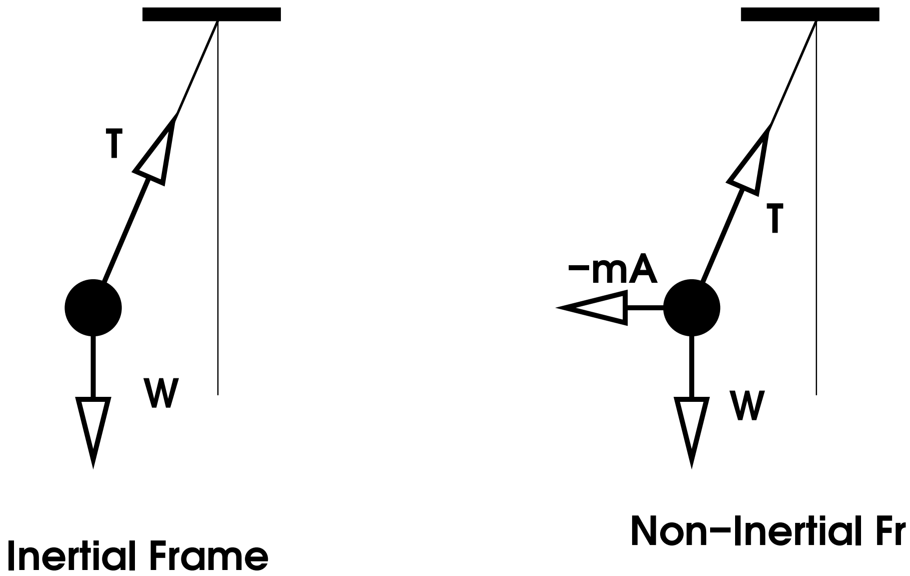
The force measured in S' be F'. Is $F' = F$ for the same arguments

Then, if S is inertial $m\vec{a} = F$. This implies

$$m\vec{a}' \;=\; \vec{F}' - mA$$

S' is not inertial. Everything would seem alright if a "Fictitious" force $-mA$ is considered. principle of equivalence.

Example



A car is moving with an acceleration A to the right. A pendulum is hung from the roof of the car. In inertial frame the bob is moving with an acceleration A. Passenger in the car sees the bob hanging steadily at an angle to the vertical.

Rotating Frames

Consider a system moving about z axis

Coordinate Transformations

$$
\begin{aligned}
x' &= x\cos(\omega t) + y\sin(\omega t) \\
y' &= -x\sin(\omega t) + y\cos(\omega t) \\
z' &= z
\end{aligned}
$$

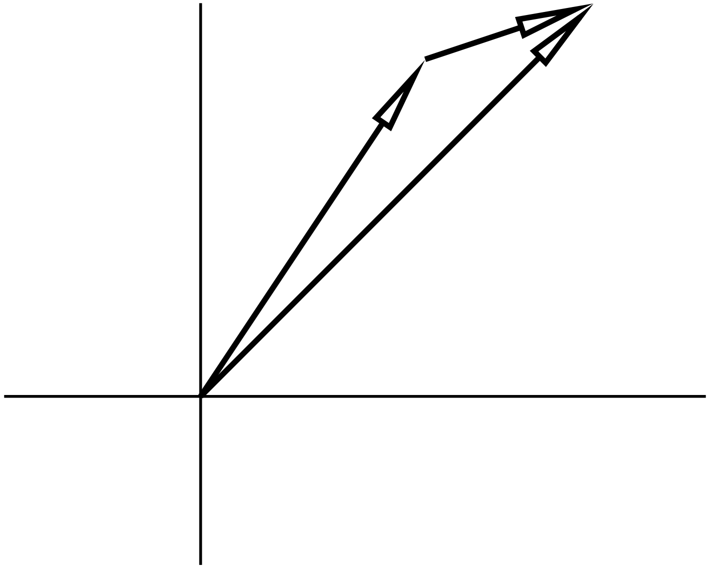
Rotating Frames

If seen from fixed system, the coordinate axes $\mathbf{i}'$ and $\mathbf{j}'$ would appear to be rotating. These vector relate to $\mathbf{i}$ and $\mathbf{j}$,

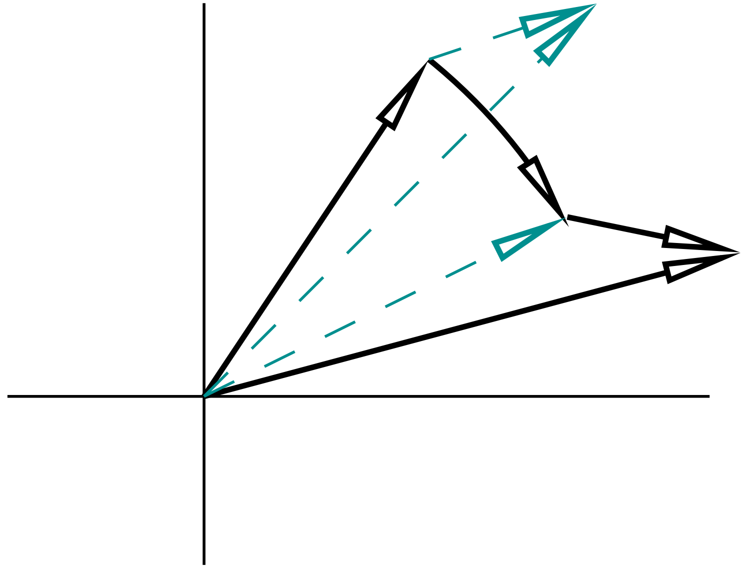
$$\begin{aligned} \mathbf{i}' &= \mathbf{i}\cos(\omega t) + \mathbf{j}\sin(\omega t) \\ \mathbf{j}' &= -\mathbf{i}\sin(\omega t) + \mathbf{j}\cos(\omega t) \\ \mathbf{k}' &= \mathbf{k} \end{aligned}$$

Rotating Frames

Suppose a vector $\vec{A}$ changes in fixed frame by amount $\Delta\vec{A}$ in time Δt.



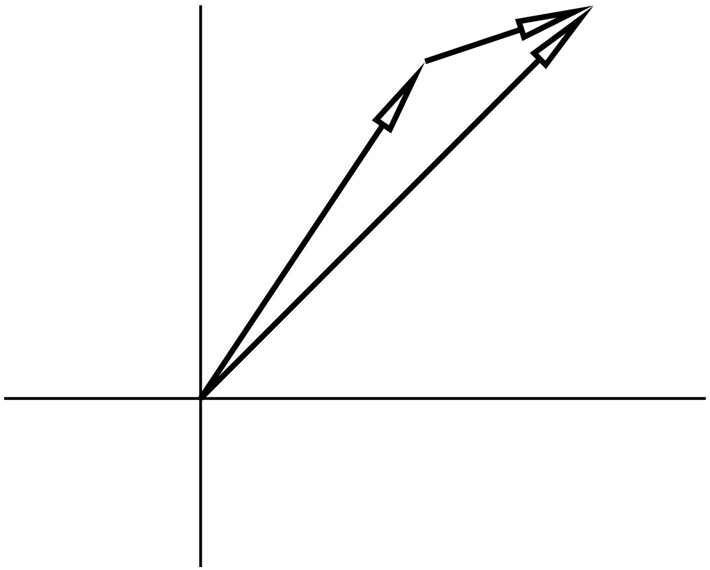
Fixed Frame



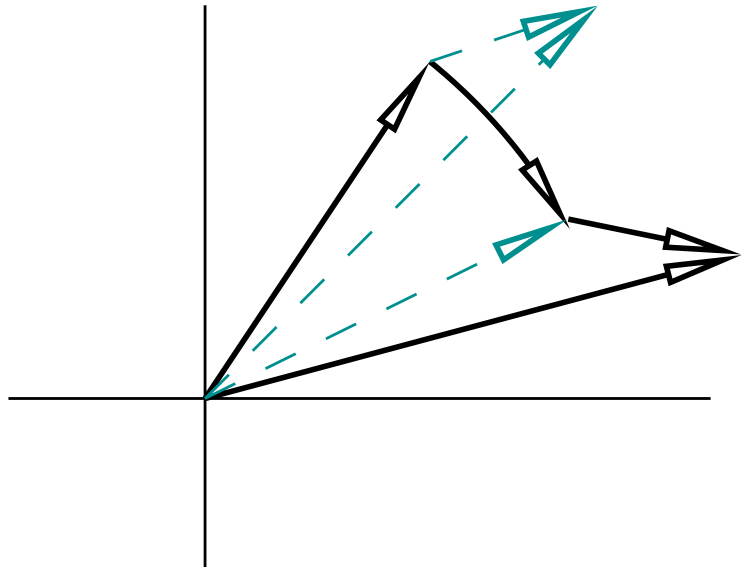
Rotating Frame

In rotating frame the change would be same, if it had occurred instantaneously. But in time Δt, the frame has turned.

Rotating Frames



Fixed Frame



Rotating Frame

$$\begin{aligned} (\Delta\vec{A})' &= \Delta\vec{A} - \vec{\omega}\times\vec{A}(\Delta t) \\ \frac{d\vec{A}}{dt} &= \left(\frac{d\vec{A}}{dt}\right)_{rot} + \vec{\omega}\times\vec{A} \end{aligned}$$

Rotating Frames

If a particle is moving in fixed frame, the velocity is given by

$$\begin{aligned} \frac{d\vec{r}}{dt} &= \left(\frac{d\vec{r}}{dt}\right)_{rot} + \vec{\omega} \times \vec{r} \\ \vec{v} &= \vec{v}' + (\omega \times \vec{r}) \end{aligned}$$

The acceleration is given by

$$\begin{aligned} \frac{d\vec{v}}{dt} &= \left(\frac{d\vec{v}}{dt}\right)_{rot} + \vec{\omega} \times \vec{v} \\ \frac{d\vec{v}}{dt} &= \left(\frac{d\vec{v}'}{dt}\right)_{rot} + 2\vec{\omega} \times \vec{v}' + \vec{\omega} \times (\vec{\omega} \times \vec{r}) \end{aligned}$$

Rotating Frames

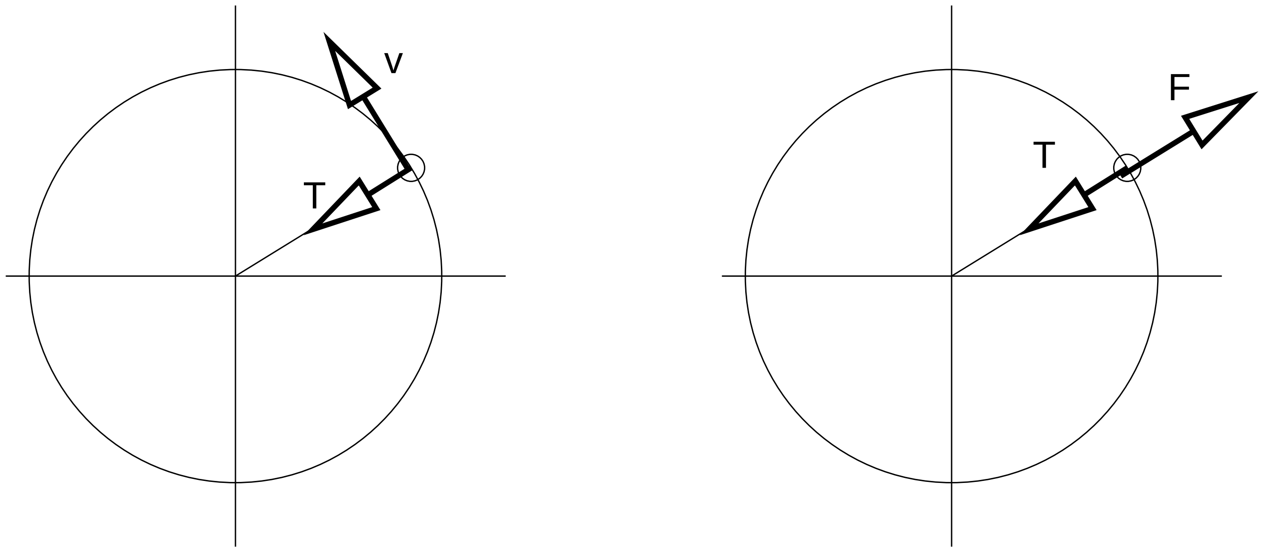
$$m\vec{a}_{rot} = m\vec{a} - 2\vec{\omega} \times \vec{v}' - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

$$m\vec{a}_{rot} = F - 2\vec{\omega} \times \vec{v}' - \vec{\omega} \times (\vec{\omega} \times \vec{r})$$

The two terms are called Coriolis Force and Centrifugal Forces.

Example

Consider a particle that is performing uniform circular motion in fixed frame with angular speed ω in a plane. A frame rotating with same angular speed will see the particle at rest. The free body diagrams are



Fixed Frame

Rotating Frame