PH101 PHYSICS1

Lecture 1

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Syllabus

Topics

- Classical Mechanics
- Relativistic Mechanics
- Quantum Mechanics

Texts

- An Intro to Mechanics – Kleppner/Kolenkow
- Modern Physics – Krane

Evaluations

- Two Quizzes each of 10% weightage
- Mid-semester Exam of 30% weightage
- End-Semester Exam of 50% weightage

Kinematics in One Dimensions

- The motion of the particle is described specifying the position as a function of time, say, $x\left(t\right)$.
- The instantaneous velocity is defined as

(1) $$v\left(t\right)=\frac{dx}{dt}$$

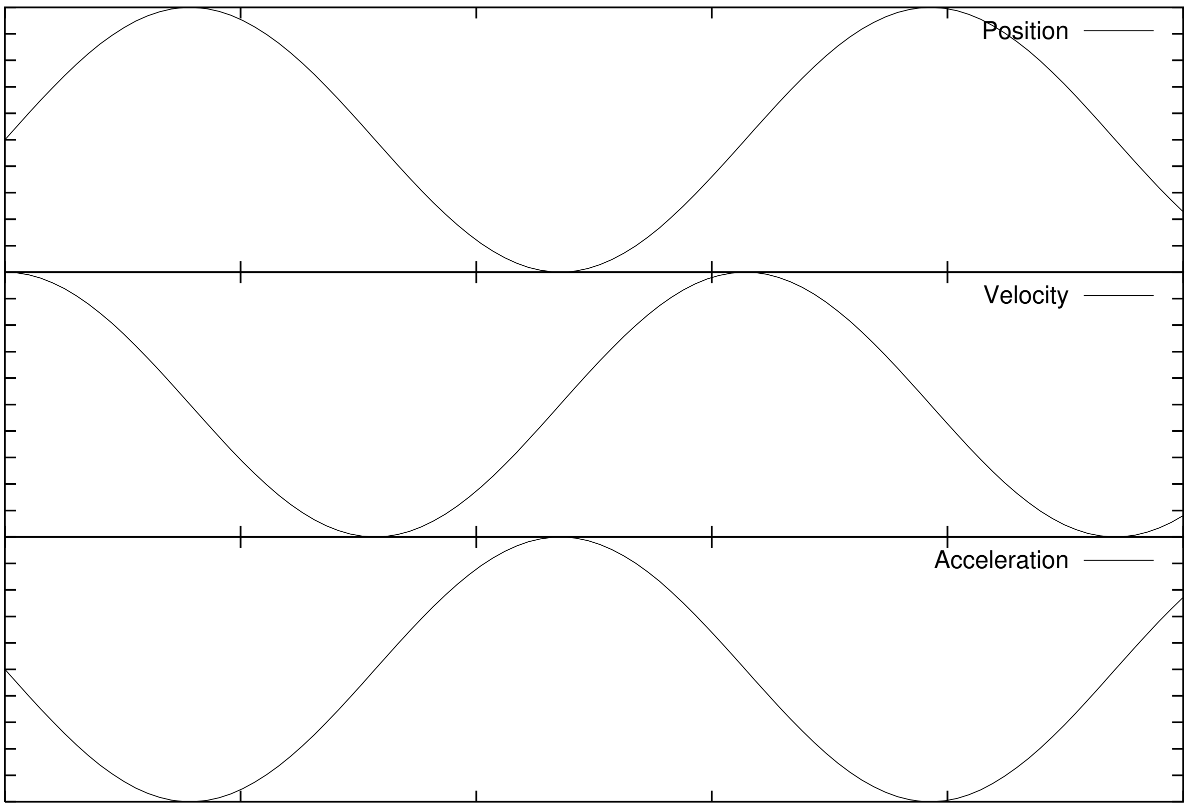
- and instantaneous acceleration, as

(2) $$a\left(t\right)=\frac{dv}{dt}=\frac{d^2x}{dt^2}$$

Simple Example

If $x(t) = \sin(t)$, then $v(t) = \cos(t)$ and $a(t) = -sin(t)$.

1D Motion



Kinematics in One Dimension

- Usually the $x(t)$ is not known in advance!
- But the acceleration $a(t)$ is known and at some given time, say t_0, position $x(t_0)$ and velocity $v(t_0)$ are known.
- The formal solution to this problem is

$$
\begin{aligned}
v(t) &= v(t_0) + \int_{t_0}^{t} a(t')\, dt' \\
x(t) &= x(t_0) + \int_{t_0}^{t} v(t')\, dt'
\end{aligned}
$$

Constant Acceleration

Let the acceleration of a particle be a_0, a constant at all times. If, at $t = 0$ velocity of the particle is v_0, then

$$\begin{aligned} v(t) &= v_0 + \int_0^t a_0 dt \\ &= v_0 + a_0 t \end{aligned}$$

And if the position at $t = 0$ is x_0,

$$x(t) = x_0 + v_0 t + \frac{1}{2} a_0 t^2 \qquad (3)$$

These are familiar formulae.

- More complex situations may arise, where an acceleration is specified as a function of position, velocity and time. $a\left(x, \dot{x}, t\right)$. In this case, we need to solve a differential equation

$$\frac{d^2x}{dt^2} = a\left(x, \dot{x}, t\right)$$

which may or may not be simple.

An Example

Acceleration of a particle is given by $a\left(x\right) = -\omega^2 x$. And at time $t = 0$, position is A and velocity is zero.

$$\begin{aligned} \frac{dv}{dt} &= -\omega^2 x \\ \frac{dv}{dx}\frac{dx}{dt} &= -\omega^2 x \\ v\frac{dv}{dx} &= -\omega^2 x \end{aligned}$$

After integrating this, we get

$$v = \omega\sqrt{A^2 - x^2} \tag{4}$$

An Example

(5) $$\frac{dx}{dt} = \omega\sqrt{A^2 - x^2}$$

Finally the solution for x is

(6) $$x(t) = A\sin(\omega t)$$

We have considered this motion earlier.

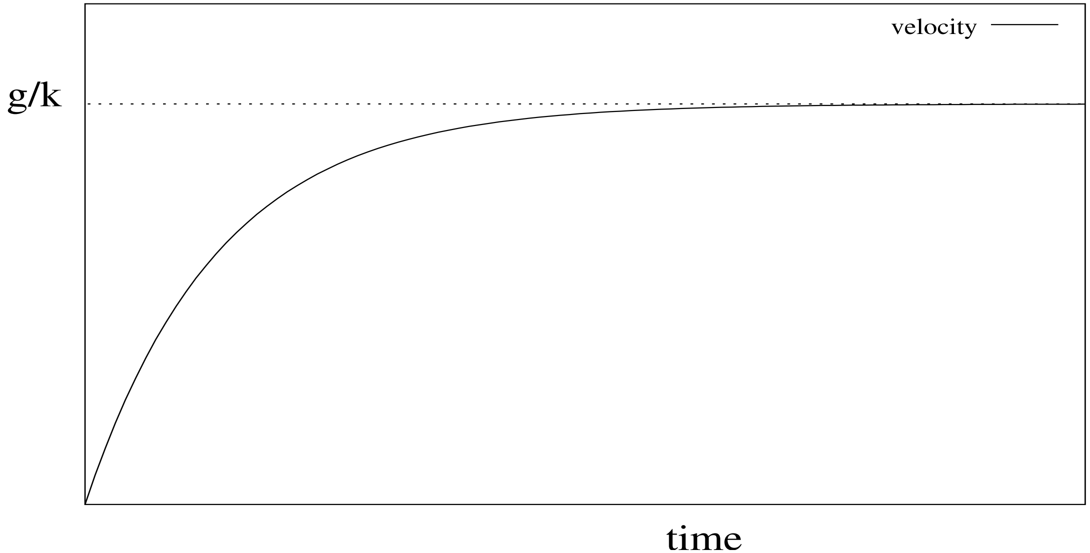
Air Resistance

Suppose a ball is falling under gravity in air, resistance of which is proportional to the velocity of the ball.

$$a(\dot{y}) = -g - k\dot{y}$$

If the ball was just dropped, velocity of the ball after time then

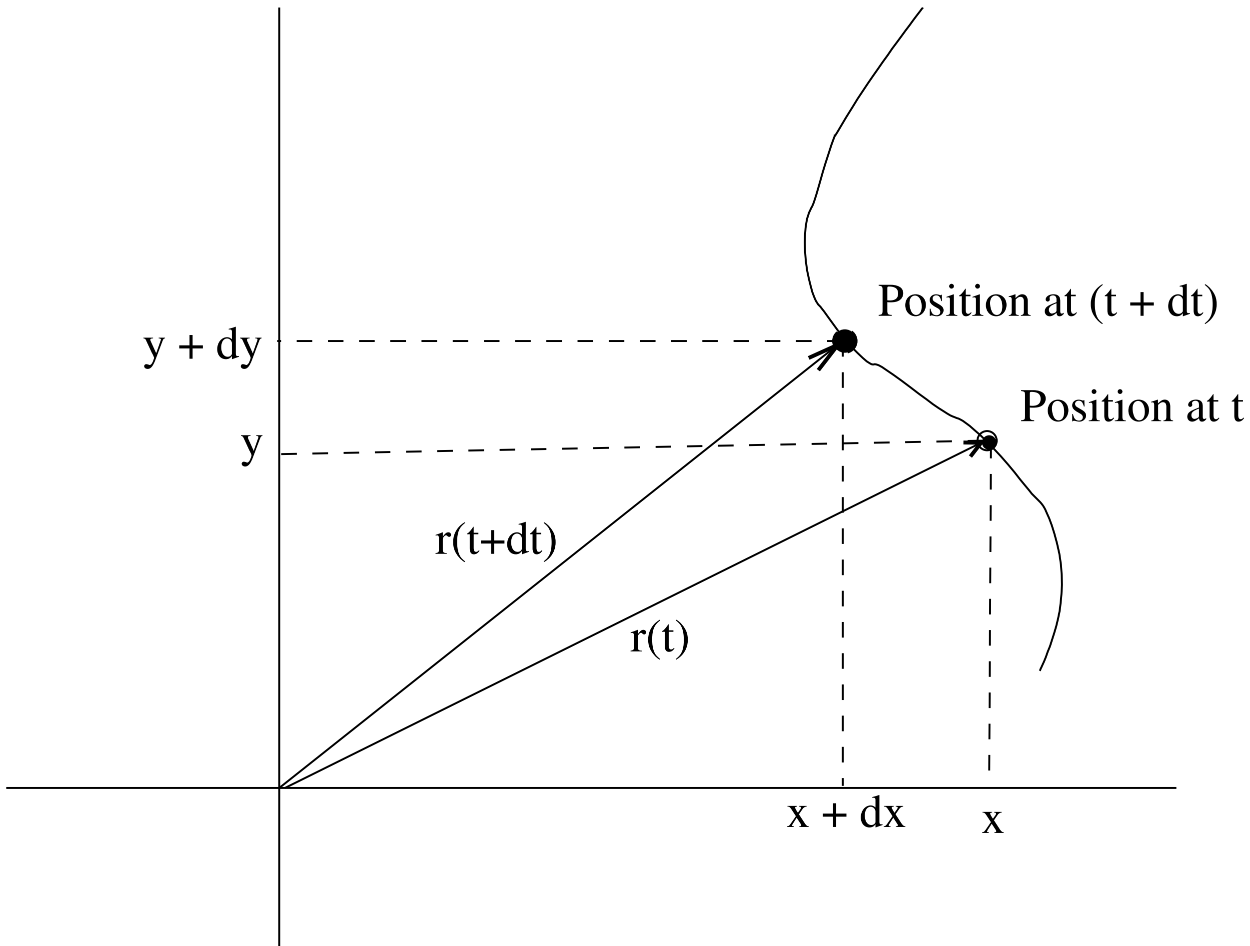
$$v(t) = -\frac{g}{k}\left(1 - e^{-kt}\right)$$



Kinematics in Two Dimensions

- The position now is specified by a vector in a plane.

$$\mathbf{r}(t) = x(t)\,\mathbf{i} + y(t)\,\mathbf{j}.$$



Kinematics in Two Dimensions

- The instantaneous velocity vector is defined as

$$\begin{aligned}\mathbf{v}(t) &= \frac{d}{dt}\mathbf{r} \\ &= \lim_{dt\to 0}\frac{\mathbf{r}(t+dt)-\mathbf{r}(t)}{dt} \\ &= \lim_{dt\to 0}\frac{x(t+dt)-x(t)}{dt}\mathbf{i} + \lim_{dt\to 0}\frac{y(t+dt)-y(t)}{dt}\mathbf{j} \\ &= \frac{dx}{dt}\mathbf{i} + \frac{dy}{dt}\mathbf{j}\end{aligned}$$

- Simillarly, we can show that

$$\mathbf{a}(t) = \frac{d^2x}{dt^2}\mathbf{i} + \frac{d^2y}{dt^2}\mathbf{j}$$

Kinematics in Two Dimensions

- Velocity and Acceleration are vector quantities.
- Formal Solutions can be written in exactly the same way as in case of one dimensional motion.
- Typical problem, however, may specify acceleration as a function of coordinates, velocities etc.
- In this case, we have to solve two differential equations

$$\begin{aligned} \frac{d^2x}{dt^2} &= a_x \\ \frac{d^2y}{dt^2} &= a_y \end{aligned}$$

Projectile Motion

A ball is projected at an angle θ with a speed u. The net acceleration is in downward direction. Then $a_x = 0$ and $a_y = -g$. The equations are

$$\begin{aligned} \frac{d^2x}{dt^2} &= 0 \\ \frac{d^2y}{dt^2} &= -g \end{aligned}$$

We know the solutions

Charged Particle in Magnetic Field

A particle has a velocity v in XY plane. Magnetic field is in z direction The acceleration is given by $\frac{q}{m}\mathbf{v} \times \mathbf{B}$

$$\begin{aligned} \frac{d^2x}{dt^2} &= \frac{qB}{m}v_y \\ \frac{d^2y}{dt^2} &= -\frac{qB}{m}v_x \end{aligned}$$

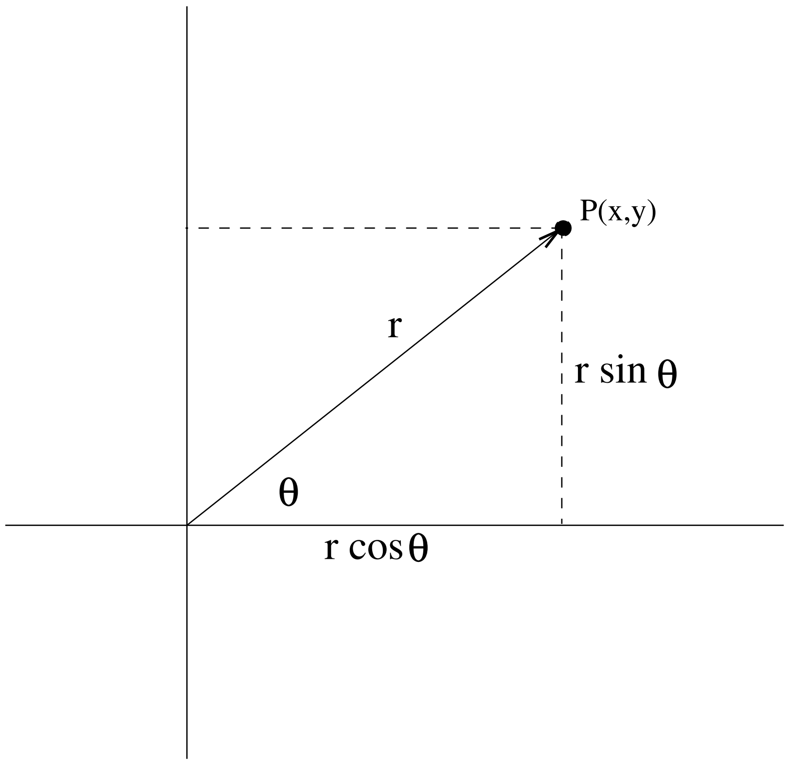
Solution is rather simple, that is circular motion in xy plane.

Polar Coordinates

Each point $P = (x, y)$
of a plane can also be specified by its distance from the origin, O and the angle that the line OP makes with the $x-$axis.

$$
\begin{aligned}
x &= r\cos\theta \\
y &= r\sin\theta
\end{aligned}
$$

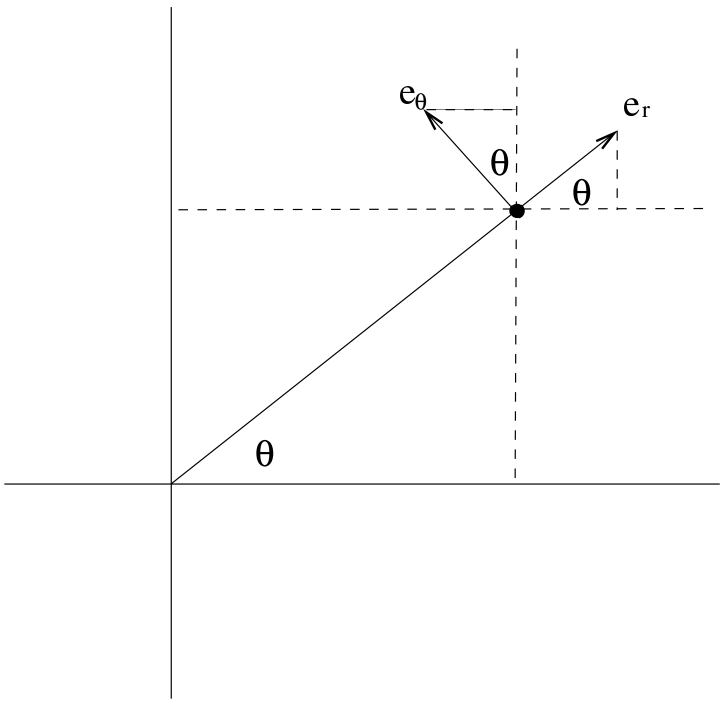
(r, θ) are called Polar Coordinates



Unit Vectors

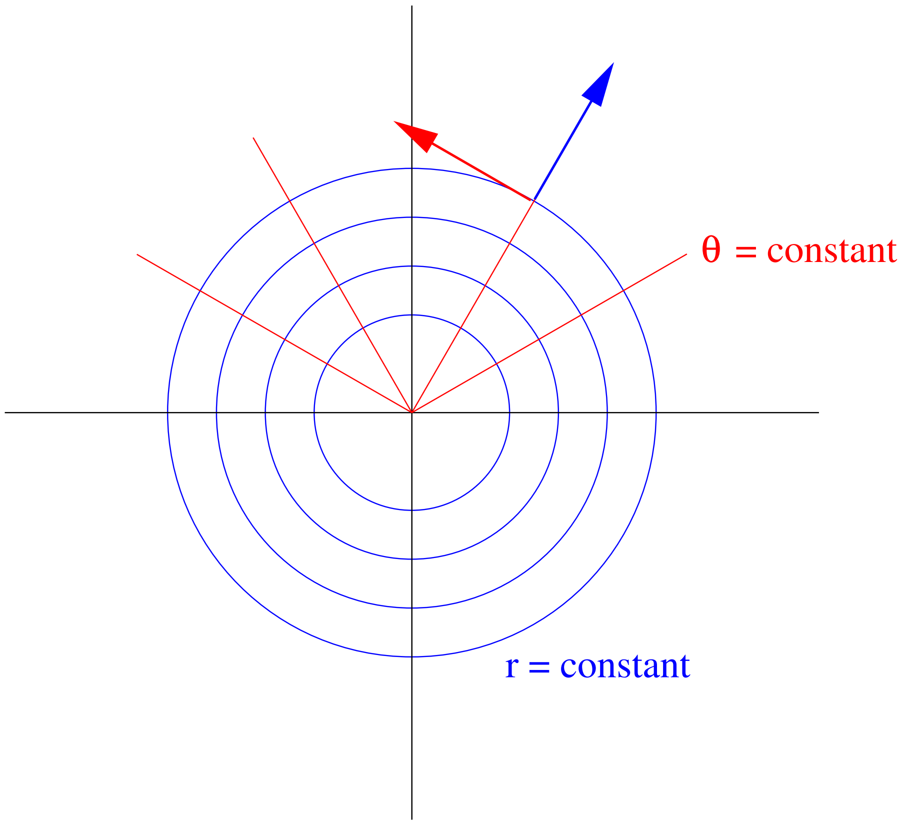
Define at each point, a set of two unit vectors $\hat{\mathbf{r}}$ and $\hat{\theta}$ as shown in the figure.

$$\begin{aligned} \hat{\mathbf{r}} &= \mathbf{i}\cos\theta + \mathbf{j}\sin\theta \\ \hat{\theta} &= -\mathbf{i}\sin\theta + \mathbf{j}\cos\theta \end{aligned}$$



Unit Vectors

There is another way of looking at these unit vectors. $\hat{\mathbf{r}}$ is a unit vector perpendicular to level surface $r = const$ and points in the direction in which r increases. Similarly for $\hat{\theta}$.



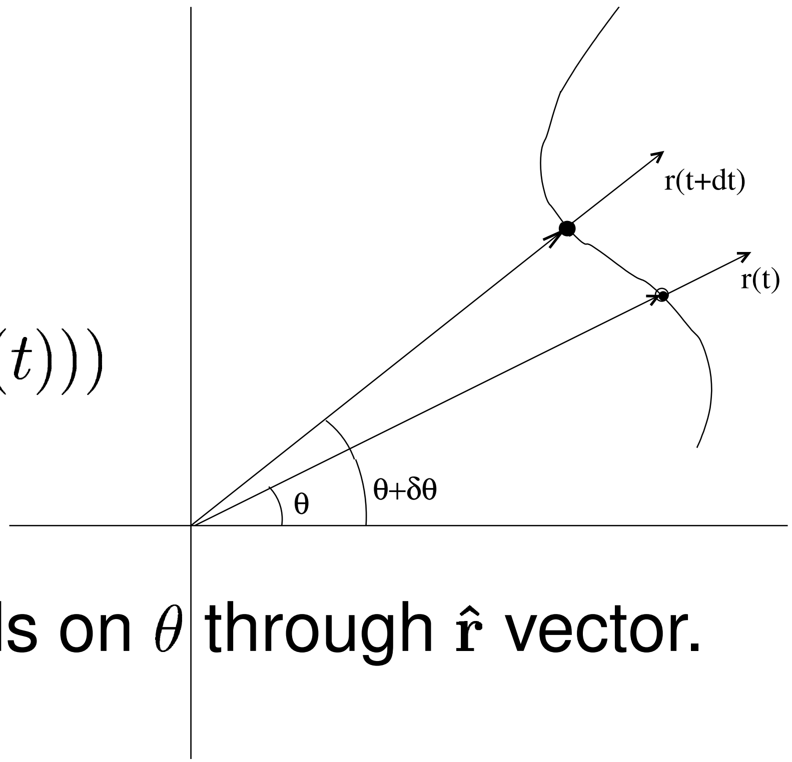
Unit Vectors

These unit vectors are functions of the polar coordinates, only of θ in fact.

$$
\begin{aligned}
\frac{\partial \hat{\mathbf{r}}}{\partial \theta} &= \hat{\theta} \\
\frac{\partial \hat{\theta}}{\partial \theta} &= -\hat{\mathbf{r}}
\end{aligned}
$$

Motion in Polar Coordinates

Suppose a particle is travelling along a trajectory given by $\mathbf{r}(t)$. Now the position vector



$$\begin{aligned} \mathbf{r}(t) &= \mathbf{i}x(t) + \mathbf{j}y(t) \\ &= r(t)\left(\mathbf{i}\cos(\theta(t)) + \mathbf{j}\sin(\theta(t))\right) \\ &= r(t)\,\hat{\mathbf{r}}(\theta(t)) \end{aligned}$$

clearly the position vector depends on θ through $\hat{\mathbf{r}}$ vector.

Motion in Polar Coordinates

- Then the velocity vector

$$
\begin{aligned}
\mathbf{v} &= \frac{d\mathbf{r}}{dt} \\
&= \frac{d}{dt}\left(r\left(t\right)\hat{\mathbf{r}}\left(\theta\left(t\right)\right)\right) \\
&= \frac{dr}{dt}\hat{\mathbf{r}} + r\frac{d\hat{\mathbf{r}}}{d\theta}\frac{d\theta}{dt} \\
&= \dot{r}\hat{\mathbf{r}}+r\dot{\theta}\hat{\theta}
\end{aligned}
$$

- The two components of the velocity are called radial and tangential velocity. $\dot{\theta}$ is called the angular speed.

Motion in Polar Coordinates

- The Acceleration Vector

$$\begin{aligned}\mathbf{a} &= \frac{d\mathbf{v}}{dt} \\ &= \left(\ddot{r} - r\dot{\theta}^2\right)\hat{\mathbf{r}} + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\hat{\theta}\end{aligned}$$

- The term $-r\dot{\theta}^2\hat{\mathbf{r}}$ is usual centripetal acceleration.
- The term $2\dot{r}\dot{\theta}\hat{\theta}$ is called Coriolis acceleration.

Linear Motion

Suppose, $\theta(t) = \theta_0$. Then, $\dot{\theta} = \ddot{\theta} = 0$. All three vectors, position, velocity and acceleration are parallel.

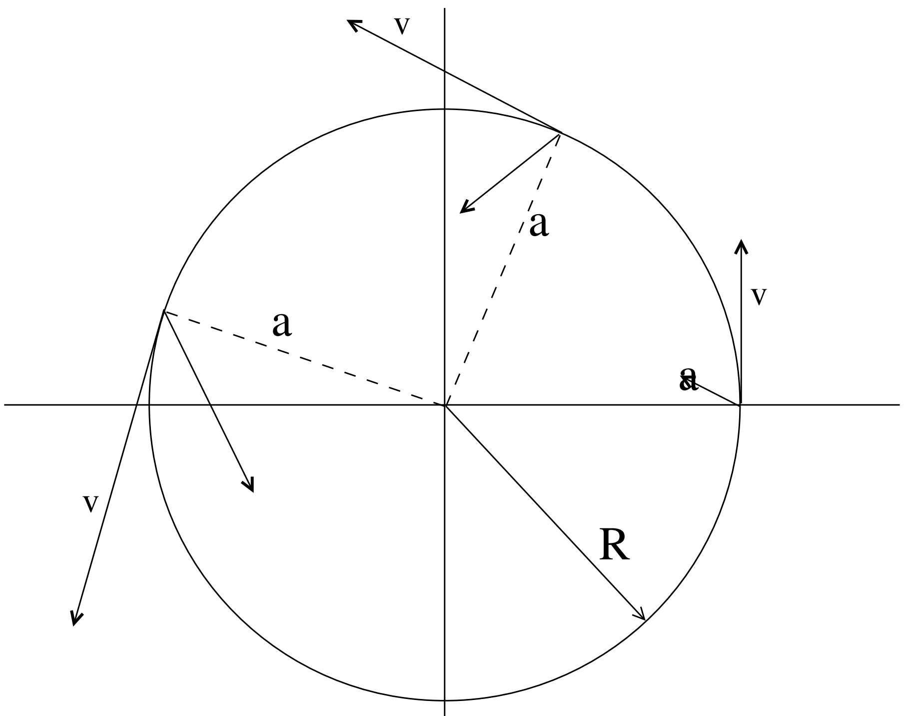
$$\begin{aligned} v &= \dot{r}\hat{\mathbf{r}} \\ a &= \ddot{r}\hat{\mathbf{r}} \end{aligned}$$

Circular Motion

In a circular motion, $r = R = Constant$. Then, $\dot{r} = \ddot{r} = 0$. Thus

$$
\begin{aligned}
v &= R\dot{\theta}\hat{\theta} \\
a &= -R\dot{\theta}^2\hat{\mathbf{r}} + R\ddot{\theta}\hat{\theta}
\end{aligned}
$$

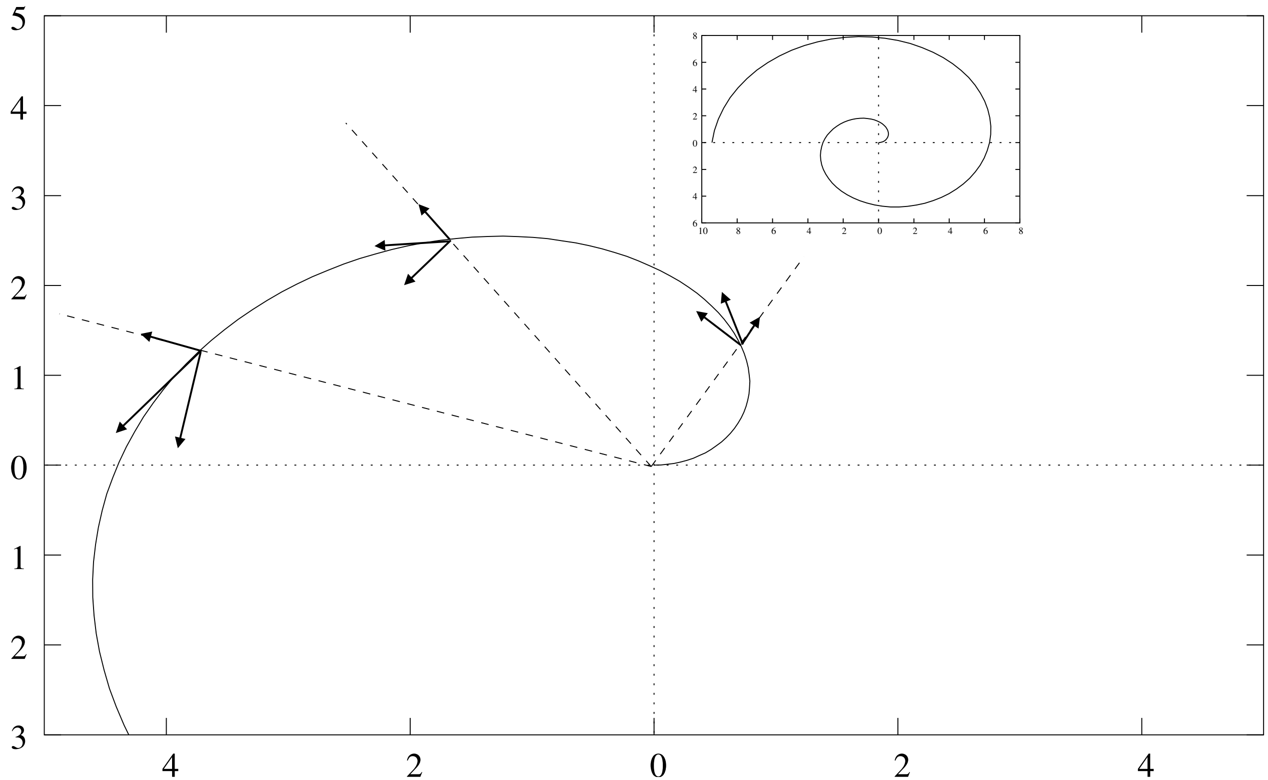
Consider a case in which $\ddot{\theta} = \alpha = constant$



Spiral Motion

Consider a particle moving on a spiral given by $r = a\theta$ with a uniform angular speed ω. Then $\dot{r} = a\dot{\theta} = a\omega$.

- $\mathbf{v} = a\omega\hat{\mathbf{r}} + a\omega^2 t\hat{\theta}$ and $\mathbf{a} = -a\omega^3 t\hat{\mathbf{r}} + 2a\omega^2\hat{\theta}$



- Central Accelerations
  When the acceleration of a particle points to the origin at all times and is only function of distance of the particle from the origin, the acceleration is called central acceleration.

$$a = f(r)\hat{\mathbf{r}}$$

- Examples are $1/r^2$, $1/r^6$, kr etc.