1) Consider a probability space (S, F, P). Show that if A and Bare in an event space (σ-field) F, so are $A \cap B$, $A \backslash B$, and $A \Delta B$.

2) Consider the random experiment of the rolling of a die. Show that $\{S, \Phi, A, B\}$ is an event space, but$\{S, \Phi, A\}$ and $\{S, \Phi, A, B, C\}$ are not event spaces.

3) Let $\Omega = \{1, 2, 3\}$. Complete $\{\{2\}, \{3\}\}$ to obtain a field. Add as few sets as possible.

4) A tower may be subjected to earthquake loads which could be of high density (event H) or of long duration (event L). It is estimated that if the load has long duration, the probability that its intensity is high is 0.7. Also, if the load has high intensity, there is 20% probability that it will be of short duration. Finally, the probability of having a long duration earthquake load is 0.3.
The designer estimated that the probability of failure when the tower is subjected to a short duration high –intensity earthquake is 0.05, whereas, this probability is doubled if the earthquake is of long duration but low intensity. Also, he is certain that the tower will fail if subjected to an earthquake with both high intensity and long duration, and that it will survive with certainty if subjected to an earthquake of low intensity and short duration.

Problems of interest are: -

Are the events H and L mutually exclusive
Are the events H and L statistically independent
Are the events H and L collectively exhaustive

Finally, calculate the probability of failure of this tower when subjected to an earthquake