

1. A smooth wire is bent into the shape of a helix, having cylindrical polar coordinates $\rho = R$ and $z = \lambda\phi$, where R and λ are constants and the z axis is vertically up. Using z as the generalized coordinate, write down the Lagrangian for a bead of mass m threaded on the wire. Find the Lagrange equation and hence the bead's vertical acceleration $\ddot{z}$. Also find its value in the limit $R \rightarrow 0$. What are your observations.
2. The Lagrangian of a system is given as, $L = \frac{1}{2}m(\dot{x}^2 + \dot{y}\dot{z} + \dot{z}^2) - \alpha(2x + 3y + z)$. Find which of the following quantities are conserved- (a) $m[2\dot{x} + \dot{z}]$, (b) $m[2\dot{x} + \dot{y} + \dot{z}]$, (c) $m[\dot{x} + \frac{3}{2}\dot{y} + \frac{1}{2}\dot{z}]$, (d) $m[2\dot{x} + 3\dot{z}]$.
3. A system has a Lagrangian given as $L = \frac{1}{2}m(\dot{q}_1^2 + \dot{q}_2^2) + M\dot{q}_1\dot{q}_2 + k\dot{q}_1\dot{q}_3$, where (q_1, q_2, q_3) are the generalised coordinates and m, M and k are positive constants. Find the coordinates which remain constant with time, and which evolve.
4. A particle of mass m is constrained to move on the inner surface of a paraboloid of revolution $x^2 + y^2 = az$ where a is a positive constant. Show that as it spirals down the surface, under the influence of gravity along z , the angular speed about the z axis is inversely proportional to z .
5. Two particles of mass m_1 and m_2 are connected by a massless thread of length l as shown in the figure. The particle of mass m_1 undergoes a circular motion with radius r_0 and angular momentum L . When a small radial displacement ϵ ($\ll r_0$) is applied its radial coordinate is found to oscillate about r_0 . Find the frequency of the oscillation.

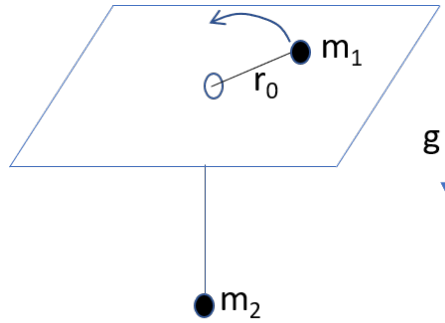


Figure 1:

6. The Lagrangian of a system in a one dimensional space is given as $L = \frac{1}{2} \left\{ \left(\frac{dx}{dt} \right)^2 - x^2 \right\}$. Find the value of the action along the path $(x = 0, t = 0)$ to $(x = x_0, t = t_0)$.
7. Consider a simple pendulum comprising a bob mass m attached to a string of length l . Use Lagrange multiplier to find the equation of motion and the force of constraint.
8. A mass m_1 rests on a frictionless horizontal table. Attached to it is a string which runs horizontally to the edge of the table, where it passes over a frictionless, small pulley and down to where it supports another mass m_2 . The distances of m_1 and m_2 from the pulley are x and y with the constraint equation as $f(x, y) = x + y = \text{const}$. Write down the two modified Lagrange equations and solve them for $\ddot{x}$, $\ddot{y}$, and the Lagrange multiplier λ . Find the tension forces on the two masses.