

# Homogenization Techniques for Granular Materials: A Critical Review

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## Abstract

Granular materials are ubiquitous in nature and are usually composed of millions of discrete particles. While numerically solving engineering scale problems with such particulate materials, either following a discrete or continuum approach, it becomes imperative to adopt a homogenization strategy to aid the computational efficiency. Homogenization techniques ensure that the underlying discreteness of the granular assembly does not introduce errors in the calculations of representative field variables of the ensemble, which is often characterized by a chosen representative volume element (RVE) at meso-level. Some of the available techniques for such homogenization are discussed in this review. This includes traditional volume averaging methods, where the stress tensor can be related to the outward-oriented contact forces of an RVE while the strain tensor is related to the displacements of the boundary particles of the RVE [1]. On the contrary, some of the authors have employed energy postulates to link the micro-macro field variables, and derived different frameworks for identifying compatible stress-strain fields at the meso-scale [2]. However, challenges remain in determination of the RVE size for such volume averaging. Additionally, differential expansions and integral transformations have also been applied to derive the meso-level field variables leading to Cosserat continuum and non-local Cosserat continuum descriptions, respectively [3]. Such theories have been well applied for different applications starting from studying mechanical behavior to mixing problems and thermal effects in granular media.

**Keywords:** Granular materials, homogenization techniques, RVE, micro-meso-macro (MMM)

## References:

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